## LECTURER1

## Limits and continuity

## Function

Defenition: $A$ function $f$ is a rule that assigns to each element $x$ in a set $A$ exactly one element, called $f(x)$, in a set $B$

## Domain and Range of a Functions

Recall the definition of a function.

Let $f: A \rightarrow B$ be a function from A to B . The set A is called the domain of f and the set B is called the codomain of f . The set $f(A)=\{f(x) \backslash x \in A\}$ is called the range of $f$

For example
Let $\mathrm{A}=\{1,2,3,4\}$ and $\mathrm{B}=\{\mathrm{v}, \mathrm{w}, \mathrm{x}, \mathrm{y}, \mathrm{z}\} \quad$, let $f: A \rightarrow B \quad$ be $\mathrm{f}=\{(1, \mathrm{w}),(2, \mathrm{y}),(3, \mathrm{y}),(4, \mathrm{z})\}$ then
(a)The domain of f is $\{1,2,3,4\}$
(b)The codomain of f is $(\mathrm{v}, \mathrm{w}, \mathrm{x}, \mathrm{y}, \mathrm{z})$
©The range of $f$ is $f(A)=\{f(1), f(2), f(3), f(4)\}=\{w, y, y, z\}=\{w, y, z\}$

## Definition

A relation is a function if and only if each object in its domain is paired with one and only one object in its range. This is not an easy definition, so let's take our time and consider a few examples.

Ex(1)
Consider the relation R .
$\mathrm{R}=(0,1),(0,2),(3,4)$ The domain is $\{0,3\}$ and the range is $\{1,2,4\}$. Note that the number 0 in the domain of $R$ is paired with two numbers
from the range, namely, 1 and 2 . Therefore, R is not a function. There is a construct, called a mapping diagram, which can be helpful in determining whether a relation is a function.

To craft a mapping diagram, first list the domain on the left, then the range on the right, then use arrows to indicate the ordered pairs in your relation, as shown in Figure 1


Figure 1. A mapping diagram for $R$.

It's clear from the mapping diagram in Figure 1 that the number 0 in the domain is being paired (mapped) with two different range objects, namely, 1 and 2. Thus, R is not a function. Let's look at another example.

## Example(2)

Is the relation described in T a function? First, the listing of the relation T.
$\mathrm{T}=(1,2),(3,2),(4,5)$ Next, construct a mapping diagram for the relation T. List the domain on the left, the range on the right, then use arrows to indicate the pairings, as shown in Figure2


Figure 2. A mapping diagram for $T$

