

LECTURER3

The limits

Assume f is defined in a neighborhood of c and let c and L be real numbers. The function f has limit L as x approaches c if, given any positive number ε , there is a positive number δ such that for all x ,

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

We write $\lim_{x \rightarrow c} f(x) = L$

The sentence $\lim_{x \rightarrow c} f(x) = L$ is read, "The limit of f of x as x approaches c equals L ." The notation means that the values $f(x)$ of the function f approach or equal L as the values of x approach (but do not equal) c .

Ex(3):

If $f(x) = \frac{x^2 - 3x + 2}{x - 2}$, $x \neq 2$ find $\lim_{x \rightarrow 2} f(x)$

Solution: $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x - 1)}{(x - 2)} = \lim_{x \rightarrow 2} (x - 1) = 2 - 1 = 1$

Limits at Infinity

We note that when the limit of a function $f(x)$ exist as approaches infinity, we write $\lim_{x \rightarrow \infty} f(x) = L$

Also, we write $\lim_{x \rightarrow +\infty} f(x) = L$ for +ive values of x and $\lim_{x \rightarrow -\infty} f(x) = L$ for

-ive values of x

For one sided and Tow sided limits, we have $\lim_{x \rightarrow \infty} f(x) = L$ iff $\lim_{x \rightarrow +\infty} f(x) = L$

and $\lim_{x \rightarrow -\infty} f(x) = L$