Statistical inference

lecture3

Chapter Two: Estimation

Properties of good estimator

Lecturer

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1-2-3 Consistency

خاصية الاتساق

<u>Def</u> let $X_1, \to X_n$ be r.v from a population with p.d.f, $f(x, \theta)$, $\theta \in \Omega$

The estimator $\hat{\theta}_n$ is called consistency estimator for θ iff:

$$\lim_{n \to \infty} E(\hat{\theta} - \theta) = 0$$

where

$$\left.\begin{aligned} &1)\ \lim_{n \to \infty} var(\hat{\theta}) = 0 \\ &2)\ \lim_{n \to \infty} \beta^2 = 0 \end{aligned}\right\} \quad 3)\ \lim_{n \to \infty} MSE(\hat{\theta}) = 0$$

<u>Example(1)</u>

let $x \sim \beta(n, \theta)$

is $\bar{x}, x_1$ consis. est. for θ ?

<u>sol</u>

$$① \lim_{n \to \infty} MSE(\bar{x}) = \lim_{n \to \infty} \frac{\theta(1-\theta)}{n}$$

$$= \theta(1-\theta) \lim_{n \to \infty} \frac{1}{n}$$

note:

$\frac{كمية}{\infty}$ صفر

$\therefore \lim_{n \to \infty} MSE(\bar{x}) = 0$

$\therefore \bar{x}$ is consis. est. for θ

② $\lim_{n\to\infty} MSE(X_1) = \lim_{n\to\infty} \theta(1-\theta)$

$= \theta(1-\theta)$

$\therefore X_1$ is not consis. est. for θ

Example(2) IF $X \sim N(\mu, \sigma^2)$ is a r.v of size (n), i) is $\bar{X}$ consis. est. for μ ?
ii) is S^2 consis. est. for σ^2?

Sol

i) $\because E(\bar{X}) = \mu$, $var(\bar{X}) = \frac{\sigma^2}{n}$

$\lim_{n\to\infty} var(\bar{X}) = \lim_{n\to\infty} \frac{\sigma^2}{n} = 0$

$\lim_{n\to\infty} B^2(\bar{X}) = \lim_{n\to\infty} (0)^2 = 0$

$\therefore \bar{X}$ is a consis. est. for μ

ii) $S^2 = \frac{\Sigma(X_i - \bar{X})^2}{n}$

$\because E(S^2) = \frac{n-1}{n}\sigma^2$

$B^2 = [E(S^2) - \sigma^2]^2$

$= [\sigma^2 \frac{n-1}{n} - \sigma^2]^2$

$= [\cancel{\sigma^2} - \frac{\sigma^2}{n} - \cancel{\sigma^2}]^2$

$B^2 = \frac{\sigma^4}{n^2}$

$$\therefore \lim_{n\to\infty} \beta^2 = \lim_{n\to\infty} \frac{\sigma^4}{n^2}$$

$$= 0$$

$$var(S^2) = ?$$

$$\frac{nS^2}{\sigma^2} = \frac{\cancel{n}}{\sigma^2} \cdot \frac{\sum(x_i-\bar{x})^2}{\cancel{n}} \sim \chi^2(\underbrace{n-1}_{m})$$

$$\chi^2 \begin{cases} E(x) = m \\ Var(x) = 2m \end{cases}$$

$$\therefore var\left(\frac{nS^2}{\sigma^2}\right) = 2(n-1)$$

$$\therefore var(S^2) = \frac{2(n-1)}{n^2}\sigma^4$$

$$\therefore \lim_{n\to\infty} var(S^2) = \lim_{n\to\infty} \frac{2(n-1)}{n^2}\sigma^4$$

$$= \sigma^4 \lim_{n\to\infty}\left(\frac{2n}{n^2} - \frac{2}{n}\right)$$

$$= 0$$

$\therefore$ S^2 is consis. est. for σ^2

<u>Note</u> IF $X \sim N(\mu, \sigma^2)$

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

$$\left(\frac{X-\mu}{\sigma}\right)^2 \sim \chi^2_{(1)}$$

$$\sum\left(\frac{X-\mu}{\sigma}\right)^2 \sim \chi^2(n)$$

$$\frac{\sum(x_i-\bar{x})^2}{\sigma^2} + \frac{n(\bar{x}-\mu)^2}{\sigma^2} \sim \chi^2(n)$$

$$\chi^2_{(n-1)} + \chi^2_{(1)} \sim \chi^2(n)$$